## Field Rotation in Altitude over Azimuth Mounts and Its Effect on CCD Imaging – What is the Maximum Exposure?

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When planning astronomical imaging sessions with a telescope equipped with an altitude over azimuth mount, it is useful to know which regions of the sky are most subject to the effects of field rotation. Field rotation depends on the latitude of the telescope, and the direction the telescope is pointed (altitude and azimuth). With respect to latitude, field rotation is worst at the Earth's equator and non-existent at the North and South poles. With respect to azimuth pointing, field rotation is worst when pointing North or South and minimal when pointing due East or West. Field rotation is worst when pointing at a high altitude (elevation).

Field rotation is not magnified by a telescope, but directly affects image quality through moving the focal plane about the line of sight. Image streaking is then related to the size of the diagonal of the image plane (CCD or film), the exposure time and the maximum tolerable blur. As an example, related to the Meade Deep Sky Imager (DSI), assume the following parameters:

Latitude =  $42.5^{\circ}$  N or S (Boston is ~  $42.5^{\circ}$  N)

**DSI diagonal dimension** = 6 mm

**Image Quality Constraint: Maximum allowed spot motion** = 10 micrometers at the corner of the DSI focal plane

The above assumptions can be used in a calculation to plot the regions of the sky that satisfy the image quality constraint. The polar plot (Figure 1) shows three regions defined by a red a blue and a black curve. The regions allowing "good imaging" are **outside the red curve for exposures of 15 seconds or less**, **outside the blue curve for exposures less than 30 seconds**, **outside the black boundary for exposures less than 60 seconds and outside the dashed brown boundary for exposures less than 120 seconds**. Figure 2 is a sky chart produced by the Meade Autostar Suite for Boston, Massachusetts at 12:30 AM, 14 December 2004 for comparison purposes. Note that the altitude scale on the sky chart is nonlinear. Most of sky can be imaged with single 30 second exposures. Frame stacking software that automatically derotates images can allow high quality imaging without an equatorial mount. Figures 3 through 11 are exposure maps for latitudes from 0° to 80° in 10° increments.

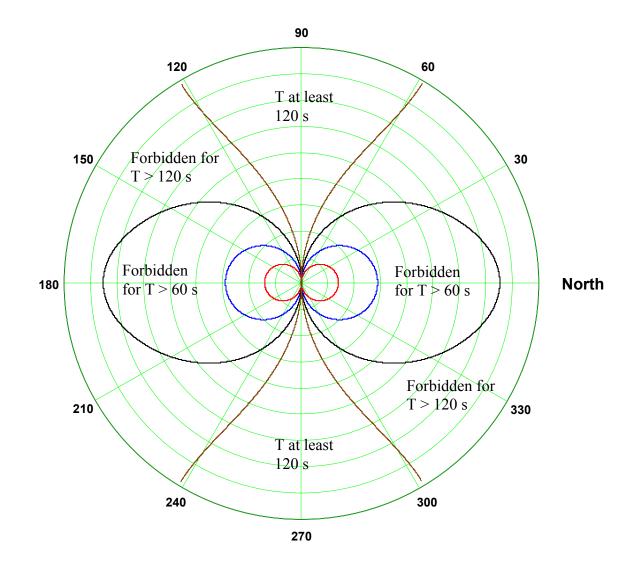


Figure 1. Sky plot showing regions of acceptable imaging (< 10 micrometers of smear at edge of field of DSI) at 42.5 degrees latitude. Local Zenith is at the center of the sky chart. INSIDE Red is forbidden for exposures greater than 15 seconds, INSIDE Blue is forbidden for exposures greater than 30 seconds and INSIDE Black is forbidden for exposures greater than 60 seconds. The dashed brown boundary defines where exposures greater than 120 seconds are forbidden. Overlay this polar plot on a sky chart that has a linear spacing in altitude. Remember the sky chart depends on location and time of day. Remember imaging at altitudes (elevations) approaching Zenith is bad and imaging due North or due South is difficult.

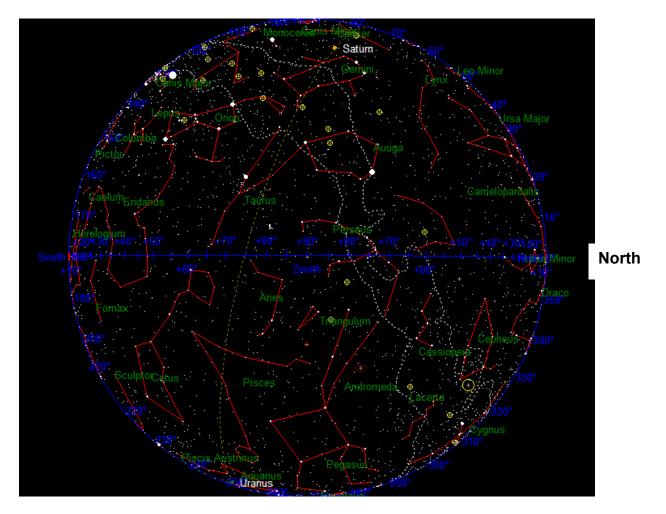


Figure 2. Sky Chart for Boston, Massachusetts, 14 December 2004, 12:30AM – Sky Chart from the *Meade Autostar Suite*. Note non-linear altitude plot.

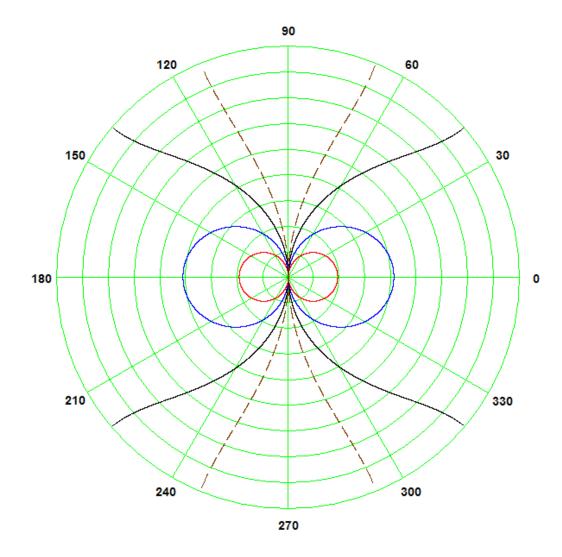
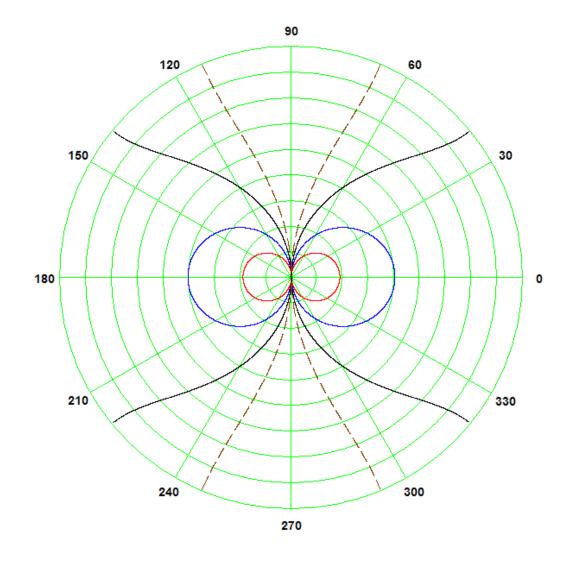
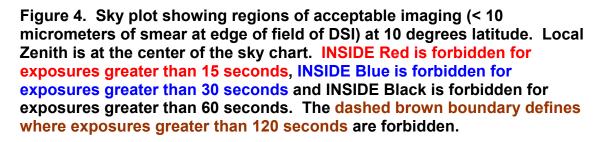


Figure 3. Sky plot showing regions of acceptable imaging (< 10 micrometers of smear at edge of field of DSI) at 0 degrees latitude. Local Zenith is at the center of the sky chart. INSIDE Red is forbidden for exposures greater than 15 seconds, INSIDE Blue is forbidden for exposures greater than 30 seconds and INSIDE Black is forbidden for exposures greater than 60 seconds. The dashed brown boundary defines where exposures greater than 120 seconds are forbidden.





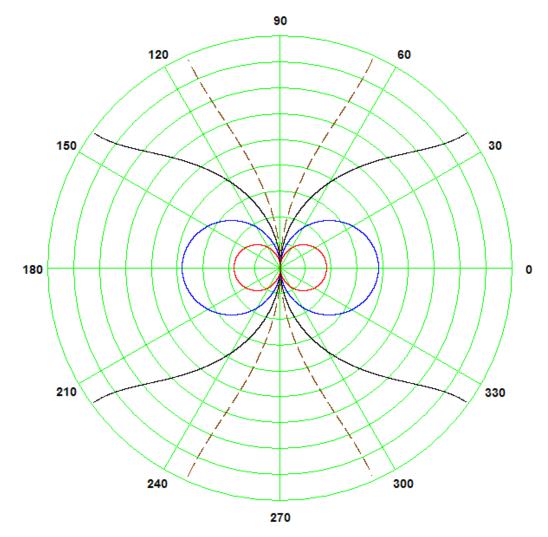


Figure 5. Sky plot showing regions of acceptable imaging (< 10 micrometers of smear at edge of field of DSI) at 20 degrees latitude. Local Zenith is at the center of the sky chart. INSIDE Red is forbidden for exposures greater than 15 seconds, INSIDE Blue is forbidden for exposures greater than 30 seconds and INSIDE Black is forbidden for exposures greater than 60 seconds. The dashed brown boundary defines where exposures greater than 120 seconds are forbidden.

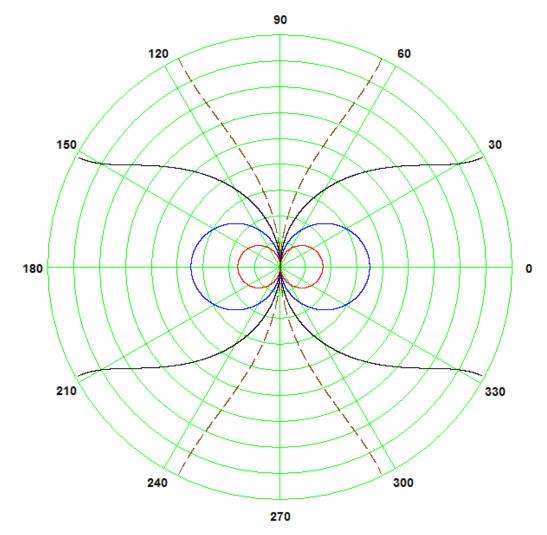


Figure 6. Sky plot showing regions of acceptable imaging (< 10 micrometers of smear at edge of field of DSI) at 30 degrees latitude. Local Zenith is at the center of the sky chart. INSIDE Red is forbidden for exposures greater than 15 seconds, INSIDE Blue is forbidden for exposures greater than 30 seconds and INSIDE Black is forbidden for exposures greater than 60 seconds. The dashed brown boundary defines where exposures greater than 120 seconds are forbidden.

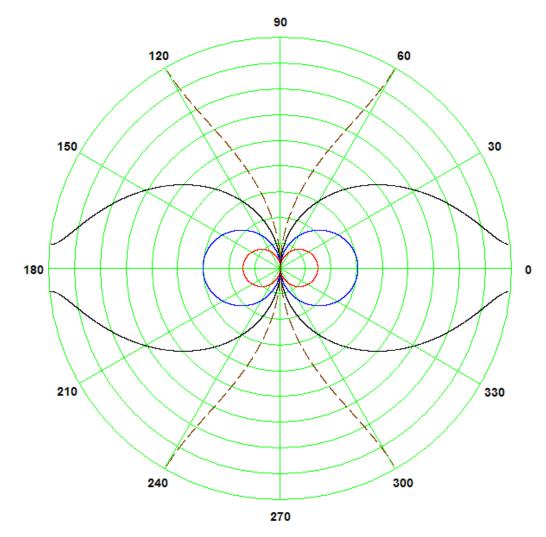


Figure 7. Sky plot showing regions of acceptable imaging (< 10 micrometers of smear at edge of field of DSI) at 40 degrees latitude. Local Zenith is at the center of the sky chart. INSIDE Red is forbidden for exposures greater than 15 seconds, INSIDE Blue is forbidden for exposures greater than 30 seconds and INSIDE Black is forbidden for exposures greater than 60 seconds. The dashed brown boundary defines where exposures greater than 120 seconds are forbidden.

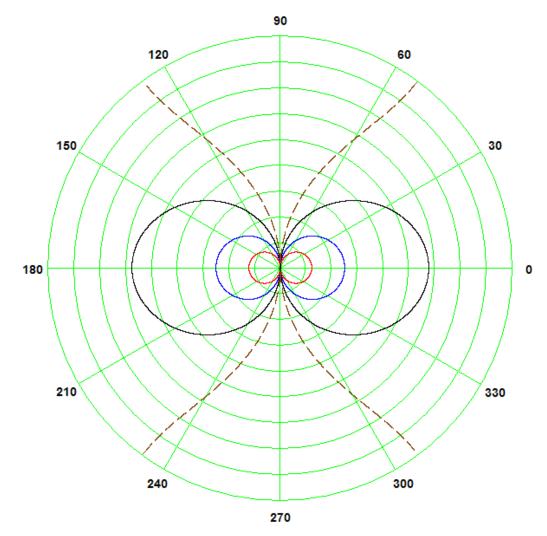


Figure 8. Sky plot showing regions of acceptable imaging (< 10 micrometers of smear at edge of field of DSI) at 50 degrees latitude. Local Zenith is at the center of the sky chart. INSIDE Red is forbidden for exposures greater than 15 seconds, INSIDE Blue is forbidden for exposures greater than 30 seconds and INSIDE Black is forbidden for exposures greater than 60 seconds. The dashed brown boundary defines where exposures greater than 120 seconds are forbidden.

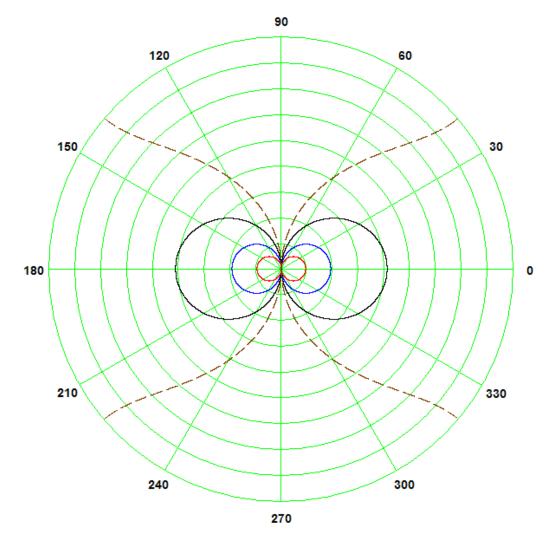


Figure 9. Sky plot showing regions of acceptable imaging (< 10 micrometers of smear at edge of field of DSI) at 60 degrees latitude. Local Zenith is at the center of the sky chart. INSIDE Red is forbidden for exposures greater than 15 seconds, INSIDE Blue is forbidden for exposures greater than 30 seconds and INSIDE Black is forbidden for exposures greater than 60 seconds. The dashed brown boundary defines where exposures greater than 120 seconds are forbidden.

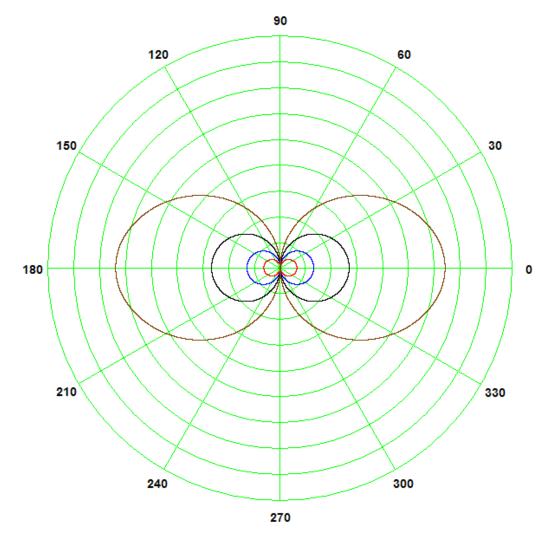


Figure 10. Sky plot showing regions of acceptable imaging (< 10 micrometers of smear at edge of field of DSI) at 70 degrees latitude. Local Zenith is at the center of the sky chart. INSIDE Red is forbidden for exposures greater than 15 seconds, INSIDE Blue is forbidden for exposures greater than 30 seconds and INSIDE Black is forbidden for exposures greater than 60 seconds. The dashed brown boundary defines where exposures greater than 120 seconds are forbidden.

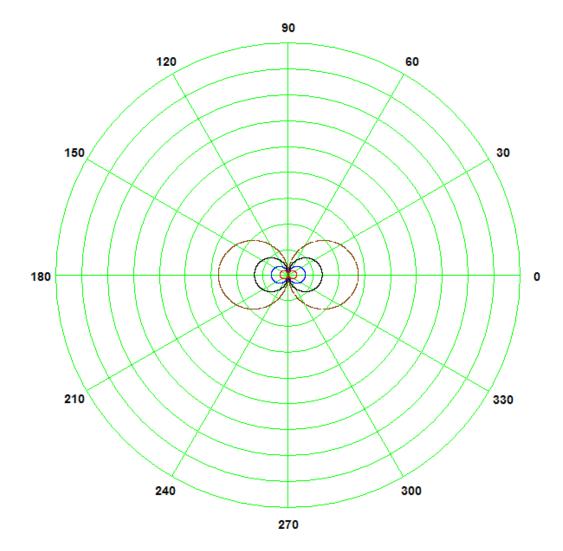


Figure 11. Sky plot showing regions of acceptable imaging (< 10 micrometers of smear at edge of field of DSI) at 80 degrees latitude. Local Zenith is at the center of the sky chart. INSIDE Red is forbidden for exposures greater than 15 seconds, INSIDE Blue is forbidden for exposures greater than 30 seconds and INSIDE Black is forbidden for exposures greater than 60 seconds. The dashed brown boundary defines where exposures greater than 120 seconds are forbidden.

## Appendix A. Mathematics of Field Rotation in an Altitude over Azimuth Mount

The origin of field rotation in an altitude over azimuth mount is due to the fact that the coordinate system of the mount is not aligned to the earth's axis of rotation. In this case, rotation of the earth couples or projects that rotation into all three axes of the telescope's coordinate system. Since an altazimuth mount typically only has two drives, rotation on the third axis goes uncompensated. When a field rotator is added to the altazimuth mount, then rotations on each axis can be removed.

The amount of field rotation depends on the amount of "misalignment" introduced by telescope pointing. The equation for field rotation in degrees per second is:

 $W = 4.178 \times 10^{-3}$  Earth rotation in degrees/sidereal second

 $\label{eq:RotationRate} RotationRate(Latitude, Azimuth, Altitude) := W \cdot cos(Latitude \cdot deg) \cdot \frac{cos(Azimuth \cdot deg)}{cos(Altitude \cdot deg)}$ 

where:

Azimuth = 0 when pointing North Azimuth = 90 degrees when pointing East, etc.

In order to calculate how far the focal plane (CCD) rotates during a given exposure, simply multiply the rotation rate in radians per second by the exposure time and then multiply that result by one half the size of the diagonal (effectively a radius) in micrometers. This blurring can be compared to size of a CCD detector (pixel), the size of the optical system's blur circle or the size of the optical blur caused by atmospheric turbulence. If we set the maximum blur caused by field rotation to, say,  $D_{max}$ , then an expression for the maximum altitude as a function of azimuth, integration time,  $D_{max}$  and latitude can be written as shown below:

$$Alt \left(Azimuth \,, \, T_{int}, \, D_{max} \,, \, Latitude\right) \coloneqq acos \left( \begin{array}{c} \frac{W \cdot cos(Latitude) \cdot \frac{Diagonal}{2} \cdot T_{int} \cdot 17.45 \cdot 10^{-3}}{2} \\ 0 \\ \hline \end{array} \right) \\ O_{max} \left( \begin{array}{c} \frac{W \cdot cos(Latitude) \cdot \frac{Diagonal}{2} \cdot T_{int} \cdot 17.45 \cdot 10^{-3}}{2} \\ 0 \\ \hline \end{array} \right) \\ \left( \begin{array}{c} \frac{W \cdot cos(Latitude) \cdot \frac{Diagonal}{2} \cdot T_{int} \cdot 17.45 \cdot 10^{-3}}{2} \\ 0 \\ \hline \end{array} \right) \\ \left( \begin{array}{c} \frac{W \cdot cos(Latitude) \cdot \frac{Diagonal}{2} \cdot T_{int} \cdot 17.45 \cdot 10^{-3}}{2} \\ 0 \\ \hline \end{array} \right) \\ \left( \begin{array}{c} \frac{W \cdot cos(Latitude) \cdot \frac{Diagonal}{2} \cdot T_{int} \cdot 17.45 \cdot 10^{-3}}{2} \\ 0 \\ \hline \end{array} \right) \\ \left( \begin{array}{c} \frac{W \cdot cos(Latitude) \cdot \frac{Diagonal}{2} \cdot T_{int} \cdot 17.45 \cdot 10^{-3}}{2} \\ 0 \\ \hline \end{array} \right) \\ \left( \begin{array}{c} \frac{W \cdot cos(Latitude) \cdot \frac{Diagonal}{2} \cdot T_{int} \cdot 17.45 \cdot 10^{-3}}{2} \\ 0 \\ \hline \end{array} \right) \\ \left( \begin{array}{c} \frac{W \cdot cos(Latitude) \cdot \frac{W \cdot cos(Latitude) \cdot 1}{2} \\ 0 \\ \hline \end{array} \right) \\ \left( \begin{array}{c} \frac{W \cdot cos(Latitude) \cdot \frac{W \cdot cos(Latitude) \cdot 1}{2} \\ 0 \\ \hline \end{array} \right) \\ \left( \begin{array}{c} \frac{W \cdot cos(Latitude) \cdot 1}{2} \\ 0 \\ \hline \end{array} \right) \\ \left( \begin{array}{c} \frac{W \cdot cos(Latitude) \cdot 1}{2} \\ 0 \\ \hline \end{array} \right) \\ \left( \begin{array}{c} \frac{W \cdot cos(Latitude) \cdot 1}{2} \\ 0 \\ \hline \end{array} \right) \\ \left( \begin{array}{c} \frac{W \cdot cos(Latitude) \cdot 1}{2} \\ 0 \\ \hline \end{array} \right) \\ \left( \begin{array}{c} \frac{W \cdot cos(Latitude) \cdot 1}{2} \\ 0 \\ \hline \end{array} \right) \\ \left( \begin{array}{c} \frac{W \cdot cos(Latitude) \cdot 1}{2} \\ 0 \\ \hline \end{array} \right) \\ \left( \begin{array}{c} \frac{W \cdot cos(Latitude) \cdot 1}{2} \\ 0 \\ \hline \end{array} \right) \\ \left( \begin{array}{c} \frac{W \cdot cos(Latitude) \cdot 1}{2} \\ 0 \\ \hline \end{array} \right) \\ \left( \begin{array}{c} \frac{W \cdot cos(Latitude) \cdot 1}{2} \\ 0 \\ \hline \end{array} \right) \\ \left( \begin{array}{c} \frac{W \cdot cos(Latitude) \cdot 1}{2} \\ 0 \\ \hline \end{array} \right) \\ \left( \begin{array}{c} \frac{W \cdot cos(Latitude) \cdot 1}{2} \\ 0 \\ \hline \end{array} \right) \\ \left( \begin{array}{c} \frac{W \cdot cos(Latitude) \cdot 1}{2} \\ 0 \\ \hline \end{array} \right) \\ \left( \begin{array}{c} \frac{W \cdot cos(Latitude) \cdot 1}{2} \\ 0 \\ \hline \end{array} \right) \\ \left( \begin{array}{c} \frac{W \cdot cos(Latitude) \cdot 1}{2} \\ 0 \\ \hline \end{array} \right) \\ \left( \begin{array}{c} \frac{W \cdot cos(Latitude) \cdot 1}{2} \\ 0 \\ \hline \end{array} \right) \\ \left( \begin{array}{c} \frac{W \cdot cos(Latitude) \cdot 1}{2} \\ 0 \\ \hline \end{array} \right) \\ \left( \begin{array}{c} \frac{W \cdot cos(Latitude) \cdot 1}{2} \\ 0 \\ \hline \end{array} \right) \\ \left( \begin{array}{c} \frac{W \cdot cos(Latitude) \cdot 1}{2} \\ 0 \\ \end{array} \right) \\ \left( \begin{array}{c} \frac{W \cdot cos(Latitude) \cdot 1}{2} \\ 0 \\ \end{array} \right) \\ \left( \begin{array}{c} \frac{W \cdot cos(Latitude) \cdot 1}{2} \\ 0 \\ \end{array} \right) \\ \left( \begin{array}{c} \frac{W \cdot cos(Latitude) \cdot 1}{2} \\ 0 \\ \end{array} \right) \\ \left( \begin{array}{c} \frac{W \cdot cos(Latitude) \cdot 1}{2} \\ 0 \\ \end{array} \right) \\ \left( \begin{array}{c} \frac{W \cdot cos(Latitude) \cdot 1}{2} \\ \end{array} \right) \\ \left( \begin{array}{c} \frac{W \cdot cos(Latiutue)$$

 $Alt(20 \cdot deg, 60, 10^{-5}, 41 \cdot deg) = 21.456 deg$ 

Note that this is for a maximum blur of 10 micrometers, what I have termed the *image quality constraint*. This equation is used in the polar plots above.